

PARAMETERIZED TASK GRAPH SCHEDULING ALGORITHM FOR COMPARING ALGORITHMIC COMPONENTS

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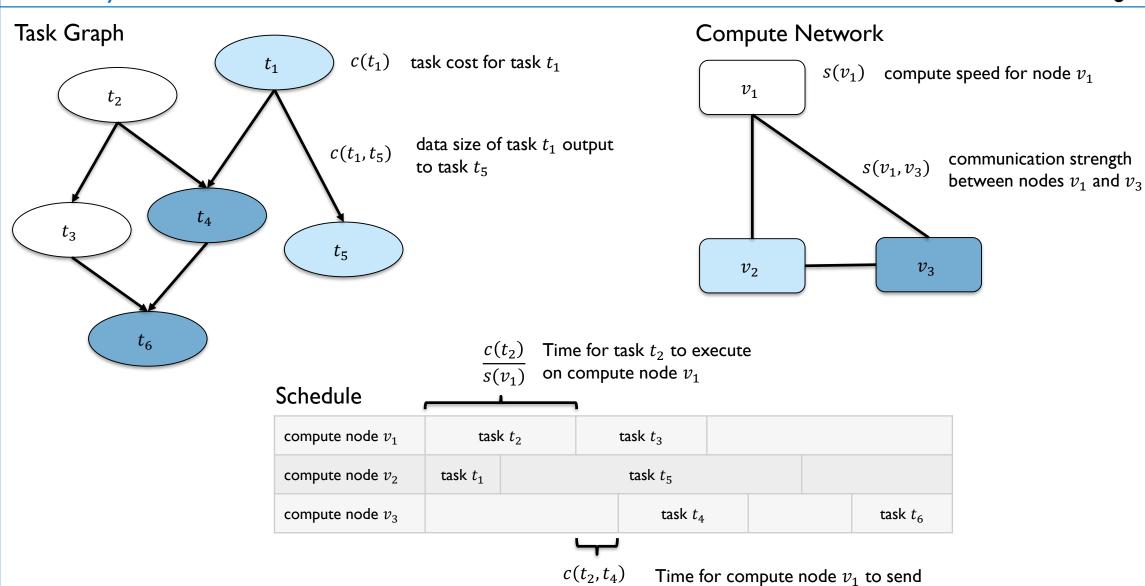
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 $s(v_1, v_3)$

Time for compute node v_1 to send task t_2 outputs to compute node v_3





A BRIEF HISTORY

- > Task scheduling work started in the 1950s
- > A survey in 1979 by Graham classified many different variants
 - » Did not consider heterogeneous communication
- > Hwang et. al. were the first to study heterogeneous communication in 1989
 - » Consider homogeneous computation
- Su et. al address fully heterogeneous case in 2019
- > Distributed computing goes mainstream tons of heuristic algorithms are proposed
 - » BIL, CPoP, ETF, FCP, FLB, GDL, HEFT, MinMin, MaxMin, MCT, MET, OLB, SMT, WBA, ...
- Modern use cases (ML/data science workflows, IoT, etc.) drive benchmarking efforts and theoretical advances
- > Bazzi et. al. show scheduling is not approximable within a constant factor in 2015





COMPARING SCHEDULING ALGORITHMS

- Scheduling is NP-Hard and not approximable, but what about for practical situations?
 - » Scheduling an ML workflow
 - » Running a scientific workflow on a supercomputing system
 - » Collecting and processing data from an IoT system
 - » Running analyses in a tactical edge environment
- Many tasks scheduling heuristic algorithms have been proposed
 - » HEFT, CPoP, BIL, ETF, FCP, FLB, GDL, MinMin, MaxMin, MCT, MET, OLB, SMT, WBA, ...
- > SAGA: Scheduling Algorithms Gathered
 - » 17 Algorithm Implementations
 - » 15 Dataset Generators
 - » Tools for Benchmarking
 - » Tools for Adversarial Analysis





COMPARING SCHEDULING ALGORITHMS

- > Many of the heuristic algorithms are similar to each other
- List Scheduling Algorithms (HEFT, CPoP, ETF, etc.):
 - 1. Compute priorities for tasks
 - 2. Greedily schedule tasks to node that minimizes/maximizes a cost function
- > Question: How do individual algorithmic differences affect algorithm performance and runtime?



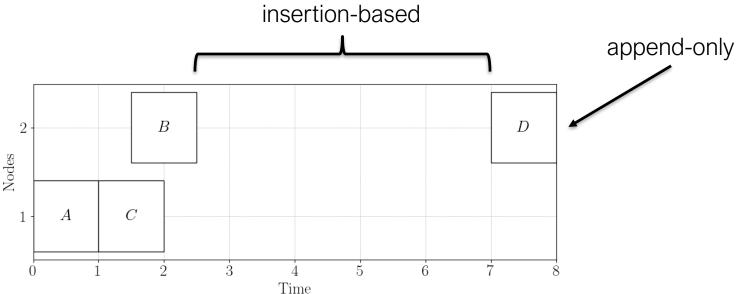


- » Algorithmic components:
 - » Priority Function: UpwardRank, CPoPRank, ArbitraryTopological
 - » Comparison Function: EFT, EST, Quickest
 - » Insertion-based vs. append-only scheduling
 - » Critical path reservation vs no reservation
 - » Sufferage vs no sufferage





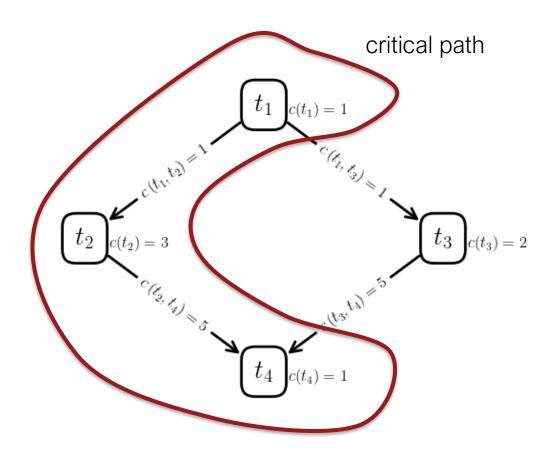
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- » 72 possible scheduling algorithms total





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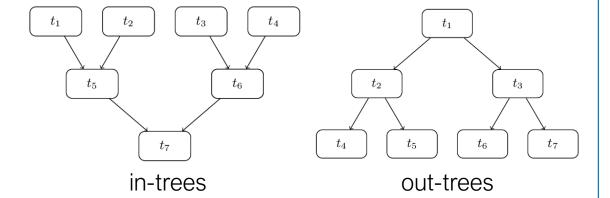


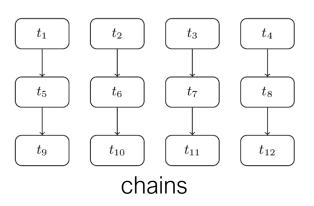


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- » 72 possible scheduling algorithms total
- » 20 datasets
 - » 4 types: in-trees, out-trees, chains, cycles

» 5 CCRs:
$$\left\{\frac{1}{5}, \frac{1}{2}, 1, 2, 5\right\}$$

- » 100 problem instances in each dataset
- » 144,000 evaluations





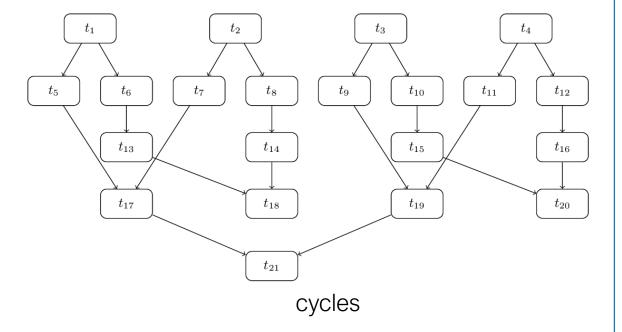




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MAKESPAN/RUNTIME RATIO

- » For a network N and task graph G
- » Let $M_A(N,G)$ denote the **makespan** of algorithm A for the problem instance (N,G)
- » The makespan ratio of algorithm A against algorithm B is

$$MR_{A,B}(N,G) = \frac{M_A(N,G)}{M_B(N,G)}$$

* how many times worse A is than B on a particular instance

» The runtime ratio of algorithm A against algorithm B is

$$RR_{A,B}(N,G) = \frac{T_A(N,G)}{T_B(N,G)}$$

* how many times longer does A take to run than B on a particular instance





PARETO-OPTIMAL SCHEDULERS

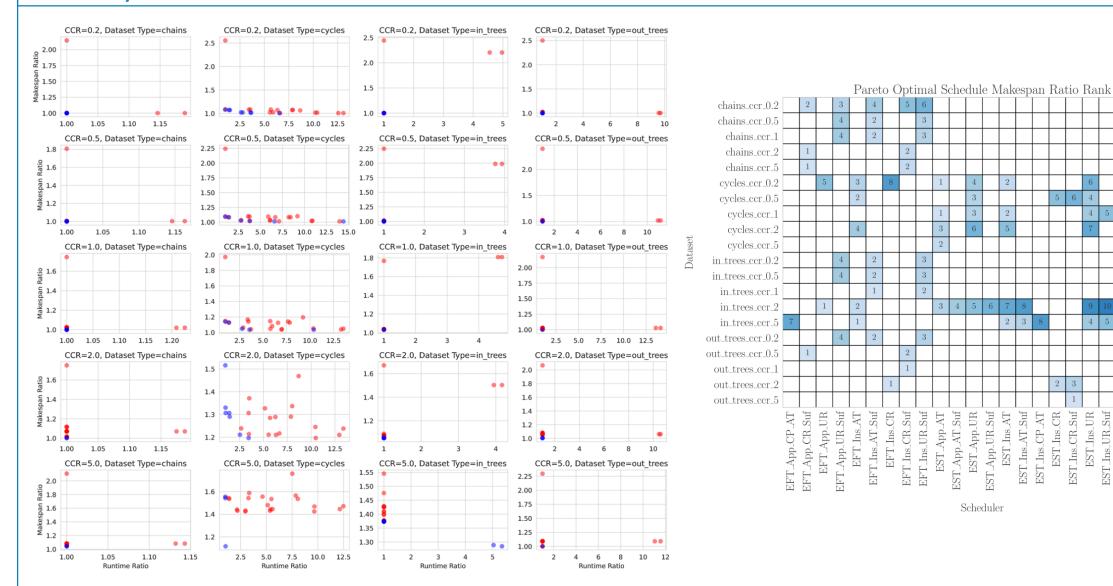
- » Pareto-optimal: for at least one dataset, no other algorithm exists that has better average makespan ratio and average runtime ratio
- » 20/24 pareto-optimal schedulers have never been studied before

component	initial_priority	append_only	compare	critical_path	sufferage
scheduler			•	_,	
EFT_Ins_AT	ArbitraryTopological	False	EFT	False	False
EFT_Ins_AT_Suf	ArbitraryTopological	False	EFT	False	True
EST_Ins_AT	ArbitraryTopological	False	EST	False	False
EST_Ins_AT_Suf	ArbitraryTopological	False	EST	False	True
EST_Ins_CP_AT	ArbitraryTopological	False	EST	True	False
MCT [10]	ArbitraryTopological	True	EFT	False	False
Sufferage [12]	ArbitraryTopological	True	EFT	False	True
EFT_App_CP_AT	ArbitraryTopological	True	EFT	True	False
EST_App_AT	ArbitraryTopological	True	EST	False	False
EST_App_AT_Suf	ArbitraryTopological	True	EST	False	True
MET [10]	ArbitraryTopological	True	Quickest	False	False
EFT_Ins_CR	CPoPRanking	False	EFT	False	False
EFT_Ins_CR_Suf	CPoPRanking	False	EFT	False	True
EST_Ins_CR	CPoPRanking	False	EST	False	False
EST_Ins_CR_Suf	CPoPRanking	False	EST	False	True
EFT_App_CR_Suf	CPoPRanking	True	EFT	False	True
HEFT [6]	UpwardRanking	False	EFT	False	False
EFT_Ins_UR_Suf	UpwardRanking	False	EFT	False	True
EST_Ins_UR	UpwardRanking	False	EST	False	False
EST_Ins_UR_Suf	UpwardRanking	False	EST	False	True
EFT_App_UR	UpwardRanking	True	EFT	False	False
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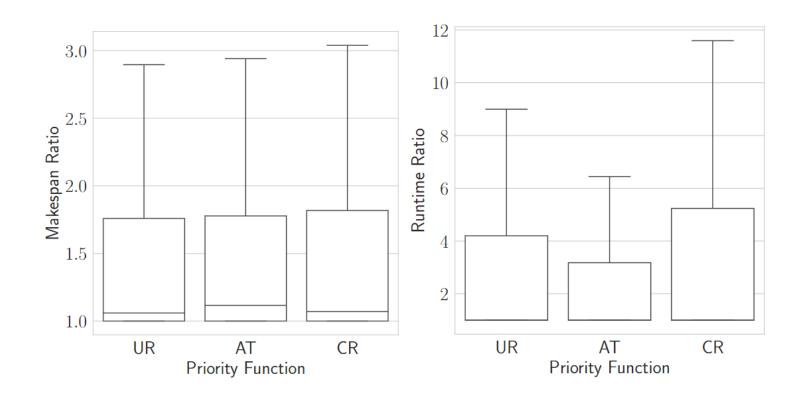
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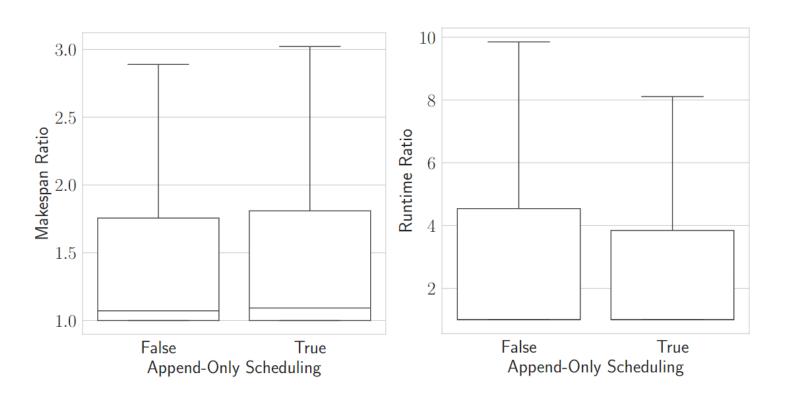
- » Priority Function
 - » UpwardRank (UR)
 - » ArbitraryTopological (AT)
 - » CPoPRank (CR)







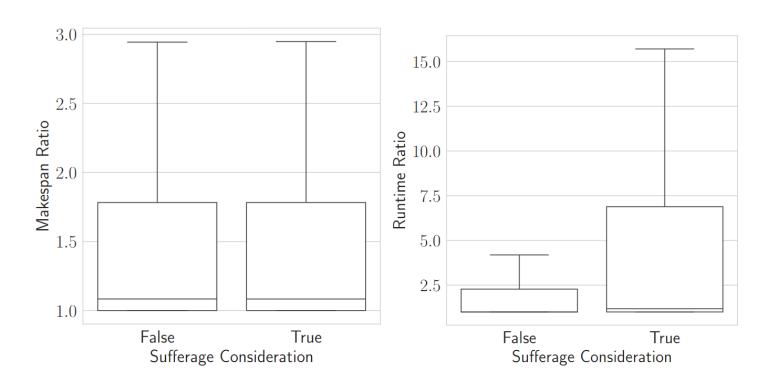
- » Priority Function
- » Append-only vs Insertion-based







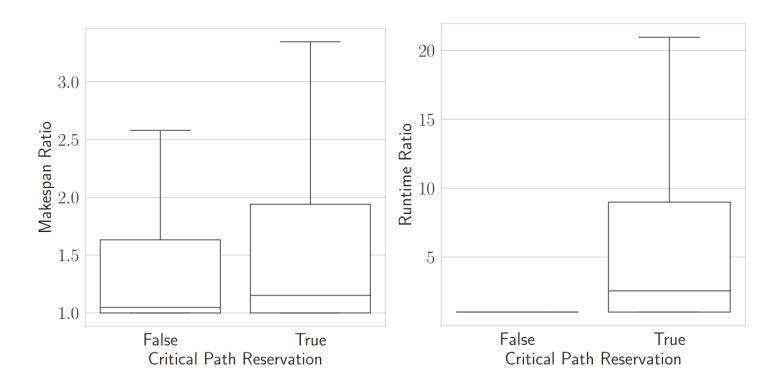
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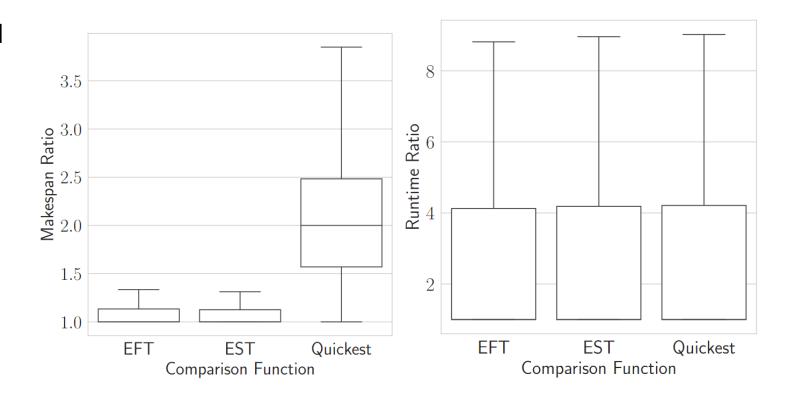
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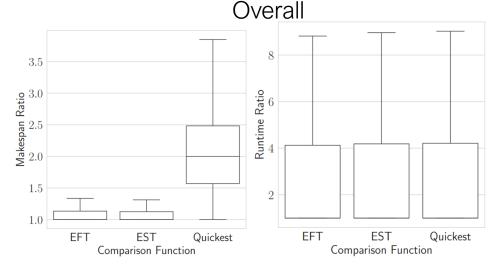
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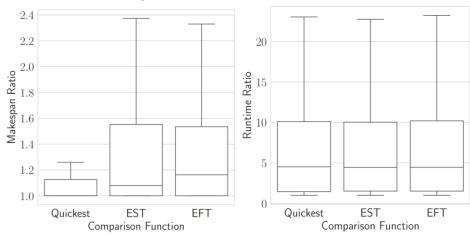




- » Priority Function
- » Append-only vs Insertion-based
- » Sufferage
- » Critical path reservation
- » Comparison Function
 - » Results look different for specific datasets!



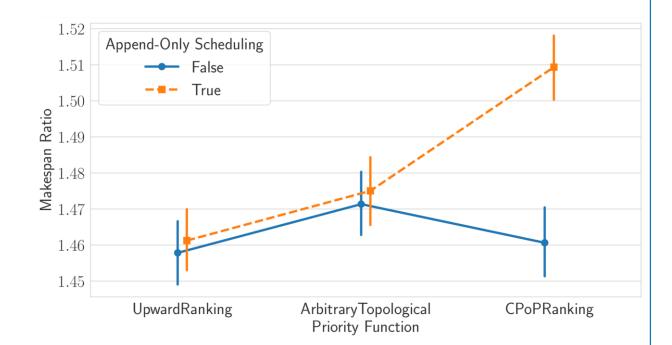
cycles dataset w/ CCR=5







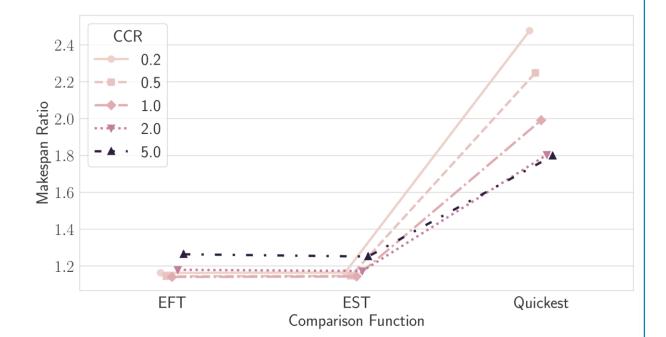
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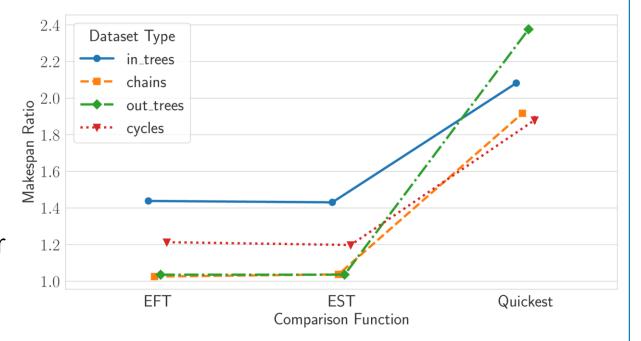
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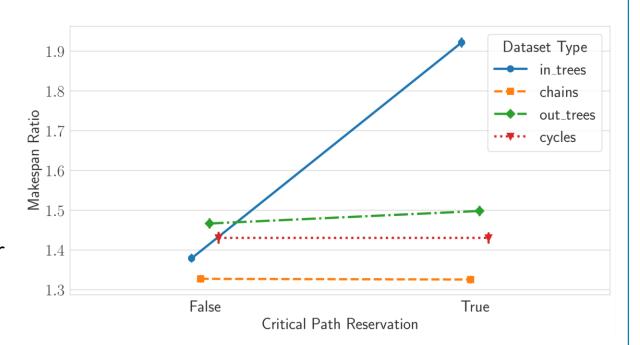
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- » Dataset Type and Critical Path
 - » Critical path reservation is particularly bad for in-tree task graph structures







CONCLUSION

- Generalized Parametric Scheduler: Proposed a modular listscheduling algorithm that enables flexible combination of five key algorithmic components.
- > **Benchmarking**: Evaluated **72 unique schedulers** generated from all component combinations on **20 datasets**, encompassing diverse graph structures and CCRs.
- > Component-level Insights: Analyzed both individual and combined effects of components on makespan and runtime across datasets.
- Dataset-specific Behavior: Revealed how component effectiveness is problem-dependent, highlighting complex interactions between components and dataset characteristics.
- > Future Work:
 - » Implement new algorithmic components (e.g., k-depth lookahead)
 - » Incorporate additional datasets
 - » Leverage findings to design adaptive or hybrid schedulers tuned to application characteristics







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